Branes at quantum criticality

JHEP07(2009)079

(http://iopscience.iop.org/1126-6708/2009/07/079)

Published by IOP Publishing for SISSA

Received: *April 14, 2009*
Revised: *June 22, 2009*
Accepted: *July 2, 2009*
Published: *July 22, 2009*

Branes at quantum criticality

J. Klusoň

Department of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37, Brno, Czech Republic

E-mail: klu@physics.muni.cz

Abstract: In this paper we propose new non-relativistic $p+1$ dimensional theory. This theory is defined in such a way that the potential term obeys the principle of detailed balance where the generating action corresponds to p-brane action. This condition ensures that the norm of the vacuum wave functional of $p+1$ dimensional theory is equal to the partition function of p-brane theory.

Keywords: D-branes

ArXiv ePrint: 0904.1343

doi:10.1088/1126-6708/2009/07/079

Contents

1 Introduction

Recently P. Hořava proposed a new intriguing formulation of theories with anisotropic scaling between time and spatial dimensions [1–4].[1] In particular, in his second paper [3] he formulated a new world-volume quantum theory of gravity and matter in $2+1$ dimensions that is strongly anisotropic between space and time in the world-volume theory. This phenomenon is well known from the study of condensed matter systems at quantum criticality [5]. Similar systems have been intensively studied from the point of view of the nonrelativistic form of the AdS/CFT correspondence [6–25, 27–34].[2] The construction of such a 2+1 dimensional theory [3] was based on the following question: Is it possible to find a quantum theory of membranes such that its ground state wave functional reproduces the partition function of the bosonic string? Generally, we can start with some equilibrium system in D dimensions that is at criticality and study how the critical behavior extends to the dynamical phenomena in $D+1$ dimensions. For example, the similar question can be asked in the context of stochastic quantization; the goal is to build a non-equilibrium system in $D+1$ dimensions that relaxes at late times to its ground state, which reproduces the partition function of a D dimensional system we are interested in.

The goal of this paper is to implement similar ideas in the case where the p dimensional system is a brane with the Nambu-Goto form of the action. We also assume that this p-brane is embedded in general D dimensional background. This assumption implies that the p dimensional action is highly non-linear with all well known consequences for the renormazibility and quantum analysis of given action. Despite of this fact we demand-with analogy with the quantum critical membrane theory [3]-that the partition function of p dimensional theory should be equivalent to the norm of the ground state of the $p+1$

[1] For recent study of cosmological aspects of these theories, see [47–49].

[2] For another approach to the study of non-relativistic systems in string theory,see for example [35–46].

dimensional theory. Note however that this correspondence is pure formal since we do not address the question whether these objects for highly nonlinear systems are really well defined. Despite of this fact we proceed further and we will see that the fundamental requirement allows us to find an action for the $p+1$ dimensional theory that is manifestly invariant under *spatial diffeomorphism* and under *rigid time translation*. Further, the resulting action obeys the *Detailed balance condition* that states that the potential term of $p+1$ dimensional theory can be derived from the variation principle of p dimensional theory.

As the next step in the construction of the action for a p-brane at criticality we extend the symmetries of given action. More precisely, we extend the time-independent spatial diffeomorphisms to all space-time diffeomorphisms that respect the preferred codimension-one foliation of $p+1$ dimensional space-time by the slices at fixed time. These diffeomorphism are known as a foliation-preserving diffeomorphism and consist a space-time dependent spatial diffeomorphism together with time-dependent reparameterization of time. Since under these transformations the original $p+1$ dimensional action is not invariant we have to introduce new gauge fields N^i and N to maintain its invariance. The presence of these new gauge fields will be crucial for the correct Hamiltonian formulation of the theory as the theory of constraint systems. In fact, since the action does not contain time derivative of N and N^i the standard analysis implies an existence of primary constraints $\pi_N \approx 0\,, \pi_i \approx 0$. The consistency of these constrains implies an existence of secondary constraints. Then we construct vacuum wave functional that is annihilated by these constraints and that is automatically the state of zero energy.

The organization of this paper is as follows. In the next section 2 we review the Lifshitz theory of D scalar fields defined on p dimensional space. In section 3 we construct the $p+1$ dimensional theory from the Nambu-Goto form of p-brane action that obeys *detailed balance condition*. In section 4 we generalize the gauge symmetries of this $p+1$-dimensional theory when we extend rigid time translation and spatial diffeomorphism to the foliation-preserving diffeomorphism. In section 5 we develop the Hamiltonian formalism for given theory and we calculate the algebra of constraints. Finally in section 6 we outline our results and suggest possible extension of this work.

2 Review of Lifshitz scalars and quantum criticality

The aim of this section is to review, following [3, 5] the physics of D free scalar fields defined on p dimensional Euclidean space with coordinates $\mathbf{x} = x^i\,, i = 1, \ldots, p$ with action

$$W = \frac{1}{2}\int d^p\mathbf{x}\delta^{ij}\partial_i\Phi^M\partial_j\Phi^N g_{MN}\,, \tag{2.1}$$

where g_{MN} is a *constant positive definite symmetric* matrix.

As in standard quantum mechanics, the fundamental object of this theory is the partition function $\mathcal{Z}$

$$\mathcal{Z} = \int \mathcal{D}\Phi(\mathbf{x}) \exp[-W(\Phi(\mathbf{x}))]\,, \tag{2.2}$$

that is defined as a path integral on the space of field configurations $\Phi^M(\mathbf{x})$. Let us assume the existence of a $p+1$ dimensional theory whose configuration space coincides with the space of all $\Phi^M(\mathbf{x})$. In other words, the wave functionals of this $p+1$-dimensional theory are functionals of $\Phi^M(\mathbf{x})$ so that $\Psi(\Phi^M(\mathbf{x}))$. Then the standard interpretation of quantum mechanics implies that $\Psi(\Phi(\mathbf{x}))\Psi^*(\Phi(\mathbf{x}))$ is a density on the configuration space. Our goal is to formulate $p+1$ dimensional system with the property that the norm of its ground-state functional $\Psi_0(\Phi(\mathbf{x}))$ reproduces the partition function (2.2)

$$\langle\Psi_0|\Psi_0\rangle = \int \mathcal{D}\Phi(\mathbf{x})\Psi_0^*(\Phi(\mathbf{x}))\Psi_0(\Phi(\mathbf{x})) = \int \mathcal{D}\Phi(\mathbf{x})\exp[-W(\Phi(\mathbf{x}))] \ . \tag{2.3}$$

To proceed further we have to introduce of Schrödinger formulation of quantum field theory. Explicitly, the basic operators in quantum field theory are $\hat{\Phi}^M(\mathbf{x})$ and $\hat{\Pi}_M(\mathbf{x})$ with canonical commutation relation

$$\left[\hat{\Phi}^M(\mathbf{x}), \hat{\Pi}_N(\mathbf{y})\right] = i\delta_N^M\delta(\mathbf{x}-\mathbf{y}) \ . \tag{2.4}$$

Further, the eigenstates of $\hat{\Phi}^M(\mathbf{x})$ are the states $|\Phi(\mathbf{x})\rangle$ that obeys

$$\hat{\Phi}^M(\mathbf{x})\left|\Phi(\mathbf{x})\right\rangle = \Phi^M(\mathbf{x})\left|\Phi(\mathbf{x})\right\rangle \ . \tag{2.5}$$

In the Schrödinger representation any state of given system is represented as the state functional $\Psi(\Phi(\mathbf{x}))$ and the action of the operator $\hat{\Phi}^M(\mathbf{x})$ on this state functional corresponds to multiplication with $\Phi^M(\mathbf{x})$. Further, the commutation relation (2.4) implies that in the Schrödinger representation the operator $\hat{\Pi}_M(\mathbf{x})$ is equal to

$$\hat{\Pi}_M(\mathbf{x}) = -i\frac{\delta}{\delta\Phi^M(\mathbf{x})} \ . \tag{2.6}$$

Let us now assume that the Hamiltonian of the $p+1$ dimensional theory has the form

$$\hat{H} = \frac{1}{2}\int d^p\mathbf{x}\hat{\mathcal{H}} \equiv \int d^p\mathbf{x}\hat{Q}_M^\dagger(\mathbf{x})\hat{g}^{MN}\hat{Q}_N(\mathbf{x}) \,, \tag{2.7}$$

where $\hat{Q}_M, \hat{Q}_M^\dagger$ are defined as

$$\hat{Q}_M = i\hat{\Pi}_M + \frac{1}{2}\frac{\delta W[\hat{\Phi}]}{\delta\hat{\Phi}^M(\mathbf{x})} \,, \qquad \hat{Q}_M^\dagger = -i\hat{\Pi}_M + \frac{1}{2}\frac{\delta W[\hat{\Phi}]}{\delta\hat{\Phi}^M(\mathbf{x})} \ . \tag{2.8}$$

Clearly, the Hamiltonian (2.7) is Hermitian and positive. Note also that $\hat{Q}_M, \hat{Q}_M^\dagger$ take the form in the Schrödinger representation

$$\hat{Q}_M = \frac{\delta}{\delta\Phi^M(\mathbf{x})} + \frac{1}{2}\frac{\delta W[\Phi]}{\delta\Phi^M(\mathbf{x})} \,, \qquad \hat{Q}_M^\dagger = -\frac{\delta}{\delta\Phi^M(\mathbf{x})} + \frac{1}{2}\frac{\delta W[\Phi]}{\delta\Phi^M(\mathbf{x})} \ . \tag{2.9}$$

Let us assume that the vacuum wave functional takes the form

$$\Psi_0(\Phi(\mathbf{x})) = \exp\left(-\frac{1}{2}W\right) = \exp\left(-\frac{1}{4}\int d^p\mathbf{x}\delta^{ij}\partial_i\Phi^M(\mathbf{x})g_{MN}\partial_j\Phi^N(\mathbf{x})\right) \ . \tag{2.10}$$

It is easy to see that $\hat{Q}_M$ defined in (2.9) annihilates Ψ_0

$$\hat{Q}_M\Psi(\Phi(\mathbf{x})) = 0 \tag{2.11}$$

as follows from the fact that

$$\frac{\delta}{\delta\Phi^M(\mathbf{x})}\Psi_0(\Phi) = -\frac{1}{2}\frac{\delta W}{\delta\Phi^M(\mathbf{x})}\Psi_0(\Phi) \ . \tag{2.12}$$

Using the definition of the Hamiltonian (2.7) we derive that the vacuum state functional is the eigenstate of the Hamiltonian of zero energy.

As the next step in the construction of the $p+1$-dimensional theory we should find corresponding Lagrangian density from the known quantum Hamiltonian (2.7). The standard procedure is to consider the classical form of this Hamiltonian when we identify $\hat{\Pi} \to \Pi$ and $\hat{\Phi} \to \Phi$ so that the classical Hamiltonian is equal to

$$\begin{aligned} H &= \frac{1}{2}\int d^p\mathbf{x}\left(-i\Pi_M(\mathbf{x}) + \frac{1}{2}\frac{\delta W}{\delta\Phi_M(\mathbf{x})}\right) g^{MN}\left(i\Pi_M(\mathbf{x}) + \frac{1}{2}\frac{\delta W}{\delta\Phi_M(\mathbf{x})}\right) \\ &= \frac{1}{2}\int d^p\mathbf{x}\left(\Pi_M(\mathbf{x})g^{MN}\Pi_N(\mathbf{x}) + \frac{1}{4}\partial_i\left[\delta^{ij}g_{MN}\partial_j\Phi^N(\mathbf{x})\right]g^{MK}\partial_i\left[\delta^{ij}g_{KL}\partial_j\Phi^L(\mathbf{x})\right]\right) \end{aligned} \tag{2.13}$$

where we used the explicit form of the variation

$$\frac{\delta W}{\delta\Phi^M(\mathbf{x})} = -\partial_i\left[\delta^{ij}g_{MN}\partial_j\Phi^N(\mathbf{x})\right] \ . \tag{2.14}$$

Then in order to find corresponding Lagrangian we use the Hamiltonian equation of motion

$$\partial_t\Phi^M(\mathbf{x}) = \left\{\Phi^M(\mathbf{x}), H\right\} = g^{MN}\Pi_N(\mathbf{x}) \tag{2.15}$$

and consequently

$$\begin{aligned} \mathcal{L} &= \partial_t\Phi^M\Pi_M - \mathcal{H} \\ &= \frac{1}{2}\partial_t\Phi^M g_{MN}\partial_t\Phi^N - \frac{1}{2}\left(\frac{1}{2}\frac{\delta W}{\delta\Phi^M}\right)g^{MN}\left(\frac{1}{2}\frac{\delta W}{\delta\Phi^N}\right) \\ &= \frac{1}{2}\partial_t\Phi^M g_{MN}\partial_t\Phi^N - \frac{1}{8}\partial_i\left[\delta^{ij}g_{MN}\partial_j\Phi^N\right]g^{MK}\partial_i\left[\delta^{ij}g_{KL}\partial_j\Phi^L\right] \ . \end{aligned} \tag{2.16}$$

We derived exactly as in [3, 5] that the Lagrangian density of the $p+1$ dimensional theory is a sum of a kinetic term that involves time derivative of Φ^M and a potential term that is derived from the variational principle. This important property is known as *detailed balance condition*.

The theory defined by the Lagrangian density (2.16) has many interesting properties. Firstly, if we define the scaling dimension of $\mathbf{x}$ as

$$[\mathbf{x}] = -1 \tag{2.17}$$

we find from the requirement of the invariance of W under scaling that the scaling dimension of Φ is

$$[\Phi] = \frac{p-2}{2} \ . \tag{2.18}$$

However then from the requirement that the scaling dimension of $p+1$ dimensional action is zero implies that the scaling dimension of t is

$$[t] = -2 \ . \tag{2.19}$$

JHEP07(2009)079

It is known from the theory of condensed matter systems that the degree of anisotropy between the time and space is measured by the dynamical critical exponent z in the sense that $[t] = -z$. Lorentz symmetry of relativistic systems imply $z = 1$ while for non-relativistic systems we have $z = 2$.

3 Branes at criticality

In this section we generalize the analysis presented in the previous section to the case when p dimensional theory is p-brane with Nambu-Goto action

$$W = \frac{1}{\kappa_W^p} \int d^p\mathbf{x} \sqrt{\det G} \, , G_{ij} = g_{MN} \partial_i \Phi^M \partial_j \Phi^N \, , \tag{3.1}$$

where the world-volume is labeled by coordinates $x^i \, , i, j = 1, \dots, p$. Further, $\Phi^M \, , M = 1, \dots, D$ are scalar fields (from the point of view of dimensional world-volume theory) and $g_{MN}(\Phi)$ is general metric of target $D-$dimensional space.

Our goal is to find $p+1$-dimensional theory with the property that the potential term obeys *detailed balance condition*. We proceed in the similar way as in previous section and demand that when we quantize this theory on $R^{p,1}$ the resulting vacuum wave functional should be equal to

$$\Psi_0[\Phi(\mathbf{x})] = \exp\left(-\frac{1}{2} W[\Phi(\mathbf{x})]\right) \, , \tag{3.2}$$

where W is given in (3.1). In other words the wave functional is function of $\Phi^M(\mathbf{x})$ that should be defined on the space of gauge orbits $\mathcal{A}/\mathcal{G}$ where $\mathcal{A}$ is a space of all fields $\Phi^M(\mathbf{x})$ and the gauge group $\mathcal{G}$ is the group of world-volume diffeomorphism. Recall that the norm of the vacuum wave functional (3.2) is equal to

$$\int_{\mathcal{A}/\mathcal{G}} \mathcal{D}\Phi[\mathbf{x}] \Psi^*[\Phi(\mathbf{x})] \Psi[\Phi(\mathbf{x})] = \int_{\mathcal{A}/\mathcal{G}} \mathcal{D}\Phi(\mathbf{x}) \exp\left(-W(\Phi(\mathbf{x}))\right) \ . \tag{3.3}$$

We see that this norm is equal to the partition function of p-brane theory that takes the form

$$Z = \int_{\mathcal{A}/\mathcal{G}} \mathcal{D}\Phi(\mathbf{x}) \exp\left(-W(\Phi(\mathbf{x}))\right) \ . \tag{3.4}$$

We should again stress that (3.3) and (3.4) are formal prescriptions since we did not precisely defined the integration measure and the space $\mathcal{A}/\mathcal{G}$.

Despite of these facts we propose a Hamiltonian of $p+1$ dimensional theory that has the property that the vacuum wave functional (3.2) is its ground state with zero energy. Following the analysis presented in previous section we introduce operators $\hat{\Phi}^M(\mathbf{x}), \hat{\Pi}_M(\mathbf{x})$ that obey the commutation relations (2.4) and define operators $\hat{Q}_M(\mathbf{x}), \hat{Q}_M^\dagger(\mathbf{x})$ as in (2.8) where now W is given in (3.1). However due to the non-linear character of (3.1) it is clear that the commutator of $\hat{Q}_M^\dagger(\mathbf{x})$ with $\hat{Q}_N(\mathbf{y})$ is nonzero and it is equal to some function of $\hat{\Phi}^M$ together with their spatial derivatives. As a consequence we have to define the quantum

Hamiltonian with the prescriptions that all $\hat{Q}^\dagger_M$'s are to the left of all $\hat{Q}'_N$. Explicitly, we propose the quantum Hamiltonian in the form

$$\begin{aligned}
\hat{H} &= \int d^p\mathbf{x}\hat{\mathcal{H}}(\mathbf{x})\,, \\
\hat{\mathcal{H}} &= \frac{\kappa^2}{2}\hat{Q}^\dagger_M\frac{g^{MN}(\hat{\Phi})}{\sqrt{\det G(\hat{\Phi})}}\hat{Q}_N = \frac{\kappa^2}{2}\left(-i\hat{\Pi}_M+\frac{\delta W}{2\delta\hat{\Phi}^M}\right)\frac{g^{MN}(\hat{\Phi})}{\sqrt{\det G(\hat{\Phi})}}\left(i\hat{\Pi}_N+\frac{\delta W}{2\delta\hat{\Phi}^N}\right)\,,
\end{aligned} \tag{3.5}$$

where κ is a coupling constant. It is easy to see that the Hamiltonian (3.5) is Hermitian and positive definite. Further, it is also clear that $\hat{Q}_M$ annihilates $\Psi_0[\Phi(\mathbf{x})]$ given in (3.2) and consequently (3.2) is a candidate for the ground state of the theory since by definition

$$\hat{H}\Psi_0[\Phi(\mathbf{x})] = 0\,. \tag{3.6}$$

Now we would like to find corresponding Lagrangian formulation of given theory defined by quantum mechanical Hamiltonian (3.5). As in previous section we consider the classical version of this Hamiltonian where we replace $\hat{\Pi}$ with Π and $\hat{\Phi}$ with Φ. It is clear that in this process we ignore the ambiguity in the ordering of Π and Φ in the Hamiltonian (3.5). With this issue in mind we claim that the classical form of the Hamiltonian density (3.5) takes the form

$$\mathcal{H} = \frac{\kappa^2}{2}\Pi_M\frac{g^{MN}}{\sqrt{\det G}}\Pi_N + \frac{\kappa^2}{8\sqrt{\det G}}\frac{\delta W}{\delta\Phi^M}g^{MN}\frac{\delta W}{\delta\Phi^N}\,. \tag{3.7}$$

Using this form of the Hamiltonian density it is easy to determine the Lagrangian density. Firstly we determine the time derivative of Φ^M from

$$\partial_t\Phi^M(\mathbf{x}) = \left\{\Phi^M(\mathbf{x}), H\right\} = \kappa^2\frac{g^{MN}\Pi_N}{\sqrt{\det G}} \tag{3.8}$$

and then we easily obtain the Lagrangian density in the form

$$\begin{aligned}
\mathcal{L} &= \partial_\tau\Phi^M\Pi_M - \mathcal{H} = \mathcal{L}_K - \mathcal{L}_V\,, \\
\mathcal{L}_K &= \frac{1}{2\kappa^2}\sqrt{\det G}\partial_\tau\Phi^M g_{MN}\partial_\tau\Phi^N\,, \\
\mathcal{L}_V &= \frac{\kappa^2}{8\kappa_W^{2p}}\sqrt{\det G}\left(\partial_M G_{ij}G^{ji} - \frac{1}{\sqrt{\det G}}\partial_i\left[g_{MK}\partial_j\Phi^K G^{ji}\sqrt{\det G}\right]\right)g^{MN} \\
&\quad\times\left(\partial_N G_{kl}G^{lk} - \frac{1}{\sqrt{\det G}}\partial_k\left[g_{NL}\partial_l\Phi^L G^{lk}\sqrt{\det G}\right]\right)\,.
\end{aligned} \tag{3.9}$$

Now we analyze the Lagrangian density (3.9) in more details. Let us consider the diffeomorphism transformations

$$x'^i = x'^i(\mathbf{x}) \tag{3.10}$$

under which the element $d^p\mathbf{x}$ transforms as

$$d^p\mathbf{x}' = d^p\mathbf{x}\,|\det D|\,, \tag{3.11}$$

JHEP07(2009)079

where we introduced $p \times p$ matrix $D^i_j = \frac{\partial x'^i}{\partial x^j}$. Further, by definition $\Phi^M(\mathbf{x})$ are world-volume scalars so that they transform under (3.10) as

$$\Phi'^M(\mathbf{x}') = \Phi^M(\mathbf{x}) \ . \tag{3.12}$$

It is easy to see that G_{ij} transform in the following way

$$\begin{aligned} G'_{ij}(\Phi'(\mathbf{x}')) &= G_{kl}(\Phi(\mathbf{x}))(D^{-1})^k_i(D^{-1})^l_j \\ \sqrt{\det G'(\Phi'(\mathbf{x}'))} &= \frac{1}{|\det D|}\sqrt{\det G(\Phi(\mathbf{x}))} \end{aligned} \tag{3.13}$$

and consequently the Lagrangian densities $\mathcal{L}_K, \mathcal{L}_V$ transform as

$$\mathcal{L}_K(\Phi'(\mathbf{x}')) = \frac{1}{|\det D(\mathbf{x})|}\mathcal{L}_K(\Phi(\mathbf{x})) \,, \qquad \mathcal{L}_V(\Phi'(\mathbf{x}')) = \frac{1}{|\det D(\mathbf{x})|}\mathcal{L}_V(\Phi(\mathbf{x})) \ . \tag{3.14}$$

Using these results we immediately obtain that $p+1$ dimensional action

$$S = \int d^p\mathbf{x} dt \mathcal{L} \tag{3.15}$$

is invariant under spatial diffeomorphisms (3.10). We could also proceed in opposite direction and demand that the $p+1$ dimensional action should be invariant under (3.10). However this requirement implies that the expression $\frac{1}{\sqrt{\det G}}$ has to be included into the Hamiltonian density (3.7). In other words, the condition that (3.2) should be annihilated by H can be also obeyed by Hamiltonian in the form $\sim \int d^p\mathbf{x}\hat{Q}^\dagger_M \hat{g}^{MN}\hat{Q}_N$ that however does not lead to diffeomorphisms invariant theory.

The additional symmetry of the action (3.15) is *global time translation*

$$t' = t + \delta t \,, \qquad \delta t = \mathrm{const}$$

as follows from the fact that

$$\Phi'^M(t', \mathbf{x}) = \Phi^M(t, \mathbf{x}) \,, \qquad \partial_{t'}\Phi'^M(t', \mathbf{x}) = \partial_t \Phi^M(t, \mathbf{x}) \ . \tag{3.16}$$

4 Foliation-preserving diffeomorphisms

We argued in previous section that the critical $(p+1)$ brane theory is invariant under *local* spatial diffeomorphisms and under *global* time translation. However it turns out that in order to take into account appropriately the fact that the $p+1$ dimensional theory is diffeomorphism invariant we have to extend these symmetries to space-time diffeomorphisms that respect the preferred codimension-one foliation $\mathcal{F}$ of world-volume theory by the slices of fixed time. After this extension we can develop the Hamiltonian formalism where the constraints related to the diffeomorphisms invariance arise in natural way.

By definition such a foliation-preserving diffeomorphisms consist space-time dependent spatial diffeomorphisms as well as time-dependent time reparameterization that are now generated by infinitesimal transformations

$$\delta x^i = x'^i - x^i = \zeta^i(t, \mathbf{x}) \,, \qquad \delta t = t' - t = f(t) \ . \tag{4.1}$$

Note also that the field Φ^M is scalar of world-volume theory and hence

$$\Phi'^M(t',\mathbf{x}') = \Phi^M(t,\mathbf{x}) \tag{4.2}$$

so that

$$\begin{aligned}
\partial_{t'}\Phi'(t',\mathbf{x}') &= \partial_t\Phi^M(t,\mathbf{x}) - \partial_t\Phi^M(t,\mathbf{x})\dot{f} - \partial_i\Phi^M(t,\mathbf{x})\dot{\zeta}^i \\
\partial_{x'^i}\Phi'^M(t',\mathbf{x}') &= \partial_i\Phi^M(t,\mathbf{x}) - \partial_j\Phi^M(t,\mathbf{x})\partial_i\zeta^j(t,\mathbf{x})
\end{aligned} \tag{4.3}$$

and we see that these objects do not transform covariantly under (4.1). Note that under such a diffeomorphism the element $dtd^p\mathbf{x}$ transforms as

$$dt'd^p\mathbf{x}' = (1+\dot{f})(1+\partial_i\zeta^i)dtd^p\mathbf{x} \ . \tag{4.4}$$

It can be also easily shown that $\sqrt{\det G}$ transforms as

$$\sqrt{\det G'(\Phi'(\mathbf{x}'))} = \sqrt{\det G(\Phi(\mathbf{x}))}(1-\partial_i\zeta^i(\mathbf{x})) \tag{4.5}$$

and consequently we find that

$$d^p\mathbf{x}'\sqrt{\det G'(\Phi'(\mathbf{x}'))} = d^p\mathbf{x}\sqrt{\det G(\Phi(\mathbf{x}))} \ . \tag{4.6}$$

However due to the fact that the time-derivative of Φ transforms non-covariantly under foliation-preserving diffeomorphism we should introduce new gauge fields N_i, N. It is convenient to derive their transformation properties under foliation-preserving diffeomorphism from relativistic diffeomorphism transformations of $p+1$ dimensional metric $g_{\mu\nu}$ by restoring the speed of light c and taking the non-relativistic limit $c\to\infty$. This procedure has been nicely reviewed in [3] where the following transformations rules for N and N^i were derived

$$\begin{aligned}
N'(t',\mathbf{x}') &= N(t,\mathbf{x})(1-\dot{f}) \, , \\
N'^i(t',\mathbf{x}') &= N^i(t,\mathbf{x}) + N^j(t,\mathbf{x})\partial_j\zeta^i(t,\mathbf{x}) - N^i(t,\mathbf{x})\dot{f} - \dot{\zeta}^i(t,\mathbf{x})
\end{aligned} \tag{4.7}$$

and consequently

$$dt'N'(t',\mathbf{x}') = dtN(t,\mathbf{x}) \ . \tag{4.8}$$

Further, the form of these transformations (4.7) suggest that it is natural to introduce following object

$$\frac{1}{N(t,\mathbf{x})}\left[\partial_t\Phi^M(t,\mathbf{x}) - N^i(t,\mathbf{x})\partial_i\Phi^M(t,\mathbf{x})\right] \tag{4.9}$$

that is invariant under folliation preserving diffeomorphism

$$\begin{aligned}
&\frac{1}{N'(t',\mathbf{x}')}\left[\partial_{t'}\Phi'^M(t',\mathbf{x}') - N'^i(t',\mathbf{x}')\partial_{i'}\Phi^M(t',\mathbf{x}')\right] = \\
&\qquad = \frac{1}{N(t,\mathbf{x})}\left[\partial_t\Phi^M(t,\mathbf{x}) - N^i(t,\mathbf{x})\partial_i\Phi^M(t,\mathbf{x})\right] \ .
\end{aligned} \tag{4.10}$$

Using these results we can finally write the $p+1$ dimensional action that is invariant under folliation-preserving diffeomorphism

$$\begin{aligned} S = & \int dt d^p \mathbf{x} N \sqrt{\det G} \left[\frac{1}{2\kappa^2} \frac{1}{N^2} (\partial_t \Phi^M - N^i \partial_i \Phi^M) g_{MN} (\partial_t \Phi^N - N^j \partial_j \Phi^N) \right. \\ & - \frac{\kappa^2}{8} \left(\partial_M G_{ij} G^{ji} - \frac{1}{\sqrt{\det G}} \partial_i \left[g_{MK} \partial_j \Phi^K G^{ji} \sqrt{\det G} \right] \right) g^{MN} \\ & \left. \times \left(\partial_N G_{kl} G^{lk} - \frac{1}{\sqrt{\det G}} \partial_k \left[g_{NL} \partial_l \Phi^L G^{lk} \sqrt{\det G} \right] \right) \right] . \end{aligned} \tag{4.11}$$

5 Hamiltonian formalism

In this section we develop the Hamiltonian formulation of the theory that is governed by the action (4.11). As the first step we introduce the momenta π_N, π_i that are conjugate to N and N^i with corresponding Poisson brackets

$$\{N(\mathbf{x}), \pi_N(\mathbf{y})\} = \delta(\mathbf{x}-\mathbf{y}) , \qquad \left\{N^i(\mathbf{x}), \pi_j(\mathbf{y})\right\} = \delta^i_j \delta(\mathbf{x}-\mathbf{y}) . \tag{5.1}$$

Then, due to the fact that the action (4.11) does not contain time derivatives of N and N^i we find that π_N and π_i are primary constraints of the theory

$$\pi_N(\mathbf{x}) \approx 0 , \qquad \pi_i(\mathbf{x}) \approx 0 . \tag{5.2}$$

As the next step we determine the momenta conjugate to $\Phi^N(\mathbf{x})$ from (4.11)

$$\Pi_N(\mathbf{x}) = \frac{1}{\kappa^2 N} \sqrt{\det G} g_{MN} (\partial_\tau \Phi^N - N^i \partial_i \Phi^N) . \tag{5.3}$$

Then using the Lagrangian density given in (4.11) we find corresponding Hamiltonian density

$$\begin{aligned} \mathcal{H} = & \partial_t \Phi^N \Pi_N - \mathcal{L} = N \left[\frac{\kappa^2}{2\sqrt{G}} \Pi^M g_{MN} \Pi^N \right. \\ & + \frac{\kappa^2}{8} \sqrt{\det G} \left(\partial_M G_{ij} G^{ji} - \frac{1}{\sqrt{\det G}} \partial_i \left[g_{MK} \partial_j \Phi^K G^{ji} \sqrt{\det G} \right] \right) g^{MN} \\ & \left. \times \left(\partial_N G_{kl} G^{lk} - \frac{1}{\sqrt{\det G}} \partial_k \left[g_{NL} \partial_l \Phi^L G^{lk} \sqrt{\det G} \right] \right) \right] + N^i \partial_i \Phi^N \Pi_N . \end{aligned} \tag{5.4}$$

Now the consistency of the primary constraints (5.2) with their time evolution implies the secondary constraints:

$$\begin{aligned} \partial_t \pi_N(\mathbf{x}) = & \{\pi_N(\mathbf{x}), H\} = - \left[\frac{\kappa^2}{2\sqrt{G}} \Pi^M g_{MN} \Pi^N - \right. \\ & - \frac{\kappa^2}{8} \sqrt{\det G} \left(\partial_M G_{ij} G^{ji} - \frac{1}{\sqrt{\det G}} \partial_i \left[g_{MK} \partial_j \Phi^K G^{ji} \sqrt{\det G} \right] \right) g^{MN} \\ & \left. \times \left(\partial_N G_{kl} G^{lk} - \frac{1}{\sqrt{\det G}} \partial_k \left[g_{NL} \partial_l \Phi^L G^{jk} \sqrt{\det G} \right] \right) \right] \equiv -T_0 \approx 0 \end{aligned} \tag{5.5}$$

JHEP07(2009)079

and

$$\partial_t \pi_i(\mathbf{x}) = \{\pi_i(\mathbf{x}), H\} = -\partial_i \Phi^N \Pi_N(\mathbf{x}) \equiv -T_i \approx 0 \tag{5.6}$$

However following [3] we suggest another class of constraints using the fact that T_0 can be written as

$$\begin{aligned} T_0 &= \frac{\kappa^2}{2} Q_M^\dagger \frac{g^{MN}}{\sqrt{\det G}} Q_N \,, \\ Q_M &= i\Pi_M + \frac{1}{2}\frac{\delta W}{\delta \Phi^M} \\ &= i\Pi_M + \frac{1}{2}\sqrt{\det G}\left[\partial_M G_{ij}\left(G^{-1}\right)^{ji} - \frac{1}{\sqrt{\det G}}\partial_i\left[g_{MN}\partial_j\Phi^N\left(G^{-1}\right)^{ji}\sqrt{\det G}\right]\right] \,. \end{aligned} \tag{5.7}$$

Then it turns out to be convenient to solve the consistency equations for π_N and π_i with collections of secondary constraints

$$Q_M \approx 0 \,, \qquad T_i \approx 0 \tag{5.8}$$

instead of T_0, T_i.

Now we would like to demonstrate that the consistency of time evolution of these constraints does not generate any additional constraints. First of all it is easy to see that Poisson brackets between T_i, Q_M and π_N, π_i vanish. Further we calculate the Poisson brackets between $T_i's$ and we find

$$\{T_i(\mathbf{x}), T_j(\mathbf{y})\} = T_i(\mathbf{x})\partial_j\delta(\mathbf{x}-\mathbf{y}) + T_j(\mathbf{x})\partial_i\delta(\mathbf{x}-\mathbf{y}) + \partial_i T_j(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) \,, \tag{5.9}$$

where we used the basic identities

$$\begin{aligned} \partial_{y^i}\delta(\mathbf{x}-\mathbf{y}) &= -\partial_{x^i}\delta(\mathbf{x}-\mathbf{y}) \,, \\ \partial_{x^i}\delta(\mathbf{x}-\mathbf{y})f(\mathbf{y}) &= \partial_i\delta(\mathbf{x}-\mathbf{y})f(\mathbf{x}) + \partial_i f(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) \,. \end{aligned} \tag{5.10}$$

Alternatively, we can introduce the smeared form of the constraints T_i when we introduce the object

$$\mathbf{T}_\zeta = \int d^p\mathbf{x}\,\zeta^i(\mathbf{x}) T_i(\mathbf{x}) \,. \tag{5.11}$$

Then the smeared form of the Poisson bracket (5.9) is

$$\{\mathbf{T}_\zeta, \mathbf{T}_\eta\} = \int d^p\mathbf{x}\left(\zeta^i\partial_i\eta^k - \eta^i\partial_i\zeta^k\right) T_k(\mathbf{x}) \,. \tag{5.12}$$

Further, let us consider the Poisson bracket of $\Pi_M(\mathbf{x})$ with any general functional $F(\Phi(\mathbf{y}))$. Using the definition of Poisson bracket

$$\{\Pi_M(\mathbf{x}), F(\Phi(\mathbf{y}))\} = -\frac{\delta F(\Phi(\mathbf{y}))}{\delta\Phi^M(\mathbf{x})} \tag{5.13}$$

and the fact that the functional derivative commute

$$\frac{\delta}{\delta\Phi^M(\mathbf{y})}\frac{\delta F}{\delta\Phi^N(\mathbf{x})} - \frac{\delta}{\delta\Phi^N(\mathbf{x})}\frac{\delta F}{\delta\Phi^M(\mathbf{y})} = 0 \tag{5.14}$$

we obtain

$$\{Q_M(\mathbf{x}),Q_N(\mathbf{y})\} = -\frac{i}{2}\frac{\delta^2 W}{\delta\Phi^M(\mathbf{x})\Phi^N(\mathbf{y})}+\frac{i}{2}\frac{\delta^2 W}{\delta\Phi^N(\mathbf{y})\delta\Phi^M(\mathbf{x})}=0$$
$$\left\{Q_M(\mathbf{x}),Q_N^\dagger(\mathbf{y})\right\} = -i\frac{\delta^2 W}{\delta\Phi^N(\mathbf{x})\Phi^M(\mathbf{y})} \ . \tag{5.15}$$

As the next step we determine the Poisson bracket between Q_M and $\mathbf{T}_\zeta$. In order to see the physical meaning of $\mathbf{T}_\zeta$ let us calculate the Poison bracket of $\mathbf{T}_\zeta$ with any function F of Φ

$$\{\mathbf{T}_\zeta,F(\Phi(\mathbf{x}))\}=-\zeta^i(\mathbf{x})\partial_i\Phi^N(\mathbf{x})=-\zeta^i(\mathbf{x})\partial_iF(\Phi(\mathbf{x})) \ . \tag{5.16}$$

This result shows that $\mathbf{T}_\zeta$ is the generator of spatial diffeomorphism $x'^i=x^i+\zeta^i(\mathbf{x})$ under which any scalar function $F(\Phi)$ transforms as

$$\delta_{\mathbf{T}_\zeta}F(\Phi(\mathbf{x}))=F(\Phi'(\mathbf{x}))-F(\Phi(\mathbf{x}))=-\zeta^i\partial_iF(\Phi(\mathbf{x}))=-\zeta^i\partial_i\Phi^N\frac{\delta F}{\delta\Phi^N(\mathbf{x})} \ . \tag{5.17}$$

We can also study the action of $\mathbf{T}_\zeta$ on more general world-volume tensors. For example, the Poisson bracket of $\mathbf{T}_\zeta$ with $G_{ij}(\mathbf{x})\equiv G_{MN}(\Phi(\mathbf{x}))\partial_i\Phi^M(\mathbf{x})\partial_j\Phi^N(\mathbf{x})$ is equal to

$$\begin{aligned}\{\mathbf{T}_\zeta,G_{ij}(\mathbf{x})\}&=-\zeta^i(\mathbf{x})\partial_k\Phi^K(\mathbf{x})\partial_KG_{MN}(\mathbf{x})\partial_k\Phi^M(\mathbf{x})\partial_j\Phi^N(\mathbf{x})\\&\quad-G_{MN}(\mathbf{x})\partial_i[\zeta^k(\mathbf{x})\partial_k\Phi^M(\mathbf{x})]\partial_j\Phi^N(\mathbf{x})-G_{MN}(\mathbf{x})\partial_i\Phi^M(\mathbf{x})\partial_j[\zeta^k(\mathbf{x})\partial_k\Phi^N(\mathbf{x})]\\&=-\zeta^i(\mathbf{x})\partial_iG_{ij}(\mathbf{x})-\partial_i\zeta^k(\mathbf{x})G_{kj}(\mathbf{x})-G_{ik}(\mathbf{x})\partial_j\zeta^k(\mathbf{x})\equiv\delta_{\mathbf{T}_\zeta}G_{ij}(\mathbf{x})\end{aligned} \tag{5.18}$$

that is clearly correct form of the variation of p dimensional metric under diffeomorphism. Then it is straightforward to calculate the Poisson bracket between $\mathbf{T}_\zeta$ and $\Pi_M(\mathbf{x})$ and $\frac{\delta W}{\delta\Phi^M(\mathbf{x})}$ and we find

$$\left\{\mathbf{T}_\zeta,\frac{\delta W}{\delta\Phi^M(\mathbf{x})}\right\} = -\zeta^i(\mathbf{x})\partial_i\left[\frac{\delta W}{\delta\Phi^M(\mathbf{x})}\right]-\frac{\delta W}{\delta\Phi^M(\mathbf{x})}\partial_i\zeta^i(\mathbf{x}) \ ,$$
$$\{\mathbf{T}_\zeta,\Pi_M(\mathbf{x})\} = -\zeta^i(\mathbf{x})\partial_i\Pi_M(\mathbf{x})-\Pi_M(\mathbf{x})\partial_i\zeta^i(\mathbf{x}) \tag{5.19}$$

and we finally obtain

$$\{\mathbf{T}_\zeta,Q_M(\mathbf{x})\}=-\partial_iQ_M(\mathbf{x})\zeta^i(\mathbf{x})-Q_M(\mathbf{x})\partial_i\zeta^i(\mathbf{x}) \ . \tag{5.20}$$

Then it is easy to see that

$$\begin{aligned}\left\{Q_M(\mathbf{x}),\int d^p\mathbf{y}NT_0(\mathbf{y})\right\} &= \frac{\kappa^2}{2}\int d^p\mathbf{y}N\left\{Q_M(\mathbf{x}),Q_P^\dagger(\mathbf{y})\right\}\frac{g^{PQ}(\mathbf{y})}{\sqrt{\det G(\mathbf{y})}}Q_Q(\mathbf{y})\\&\quad+\frac{\kappa^2}{2}\int d^p\mathbf{y}NQ_P^\dagger(\mathbf{y})\left\{Q_M(\mathbf{x}),\frac{g^{PQ}(\mathbf{y})}{\sqrt{\det G(\mathbf{y})}}\right\}Q_Q(\mathbf{y})\\&\quad+\frac{\kappa^2}{2}\int d^p\mathbf{y}NQ_P^\dagger(\mathbf{y})\frac{g^{PQ}(\mathbf{y})}{\sqrt{\det G(\mathbf{y})}}\{Q_M(\mathbf{x}),Q_Q(\mathbf{y})\}\approx 0 \ ,\end{aligned} \tag{5.21}$$

where the first and the second terms vanish on constraint surface $Q_M\approx 0$ and the third term vanishes due to the Poisson bracket (5.15). Then using (5.20) we find that

$\partial_t Q_M(\mathbf{x}) = \{Q_M(\mathbf{x}), H\} \approx 0$. In the same way we can show that the Poisson bracket between $\mathbf{T}_\zeta$ and H vanishes on constraint surface. These results imply that the consistency of time evolutions of Q_M and $\mathbf{T}_\zeta$ does not generate additional constraints.

For further purposes we also determine the Poisson bracket between $\mathbf{T}_\zeta$ and W. Using (5.20) we find

$$\begin{aligned}
\{\mathbf{T}_\zeta, W\} &= \frac{1}{2}\int d\mathbf{x}\, \{\mathbf{T}_\zeta, G_{ij}(\mathbf{x})\}\, G^{ji}(\mathbf{x})\sqrt{\det G(\mathbf{x})} \\
&= -\int d\mathbf{x} \left[\frac{1}{2}\zeta^k(\mathbf{x})\partial_k G_{ij}(\mathbf{x}) G^{ji}(\mathbf{x})\sqrt{\det G(\mathbf{x})} + \partial_i\zeta^i(\mathbf{x})\sqrt{\det G(\mathbf{x})}\right] \\
&= -\int d\mathbf{x}\partial_i \left[\zeta^i(\mathbf{x})\sqrt{\det G(\mathbf{x})}\right] = 0 \ .
\end{aligned} \tag{5.22}$$

In fact this is expected result since W is diffeomorphism invariant by construction.

Now we are ready to perform some preliminary steps in the quantization of given theory. Since the Hamiltonian is sum of the first class constrains we have to demand that each wave functional of the system should be annihilated by all these constraints. In fact, we find previously that the ground state functional

$$\Psi_0[\Phi^M(\mathbf{x})] = \exp\left[-\frac{1}{\kappa_W^p}\int d^p\mathbf{x}\sqrt{\det G}\right] \tag{5.23}$$

satisfies the constraints

$$\hat{Q}_M(\mathbf{x})\Psi_0[\Phi(\mathbf{x})] = \left(\frac{\delta}{\Phi^M(\mathbf{x})} + \frac{\delta W}{2\delta\Phi^M(\mathbf{x})}\right)\Psi_0[\Phi(\mathbf{x})] = 0 \ . \tag{5.24}$$

Further, the operator $\hat{\mathbf{T}}_\zeta$ has following form in Schrödinger representation

$$\hat{\mathbf{T}}_\zeta = -i\int d\mathbf{x}\zeta^i(\mathbf{x})\partial_i\Phi^N(\mathbf{x})\frac{\delta}{\delta\Phi^N(\mathbf{x})} \ . \tag{5.25}$$

Then it is clear that $\hat{\mathbf{T}}_\zeta$ annihilates $\Psi_0[\Phi]$ since

$$\hat{\mathbf{T}}\Psi_0[\Phi(\mathbf{x})] = i\int d\mathbf{y}\zeta^i(\mathbf{y})\partial_i\Phi^N(\mathbf{y})\frac{\delta W}{\delta\Phi(\mathbf{y})}\Psi_0[\Phi(\mathbf{x})] = 0 \tag{5.26}$$

using the fact that $\int d\mathbf{x}\zeta^i(\mathbf{x})\partial_i\Phi^N(\mathbf{x})\frac{\delta W}{\delta\Phi(\mathbf{x})}$ is equivalent to the Poisson bracket (5.22) that vanishes due to the diffeomorphism invariance of W. In other words the vacuum wave functional obeys all constraints of the theory. Further, since it is annihilated by $\hat{Q}_M$ it is also eigenstate of the Hamiltonian with zero energy. On the other hand it is an open problem whether this is a normalizable state and whether there are more general functionals that have non-zero energy with respect to given Hamiltonian.

6 Conclusion

This paper is devoted to the construction of new class of $p+1$ dimensional non-relativistic theories that obey the detailed balance condition that claims that their potential is derived

JHEP07(2009)079

from the variation principle of p dimensional Nambu-Goto form of p-brane action. We also extended symmetries of given action and construct $p+1$ dimensional action that is invariant under foliation-preserving diffeomorphisms.

We hope that these new non-relativistic theories have many interesting properties and should be studied further. In particular, it will be interesting to study their quantum properties in more details. We would also like to extend this formalism to the case of BPS and non-BPS Dp-branes and to the case of topological p-branes, following for example [50]. We also mean that it would be interesting to study the dynamics of these non-relativistic p-branes in backgrounds with the metric that does not have Euclidean signature. We hope to return to these problems in future.

Acknowledgments

This work was supported by the Czech Ministry of Education under Contract No. MSM 0021622409. I would also like to thank the Max Planck Institute at Golm for its kind hospitality during finishing of this work.

References

[1] P. Hořava, *Spectral dimension of the universe in quantum gravity at a Lifshitz point*, *Phys. Rev. Lett.* **102** (2009) 161301 [arXiv:0902.3657] [SPIRES].

[2] P. Hořava, *Quantum gravity at a Lifshitz point*, *Phys. Rev.* **D 79** (2009) 084008 [arXiv:0901.3775] [SPIRES].

[3] P. Hořava, *Membranes at quantum criticality*, *JHEP* **03** (2009) 020 [arXiv:0812.4287] [SPIRES].

[4] P. Hořava, *Quantum criticality and Yang-Mills gauge theory*, arXiv:0811.2217 [SPIRES].

[5] E. Ardonne, P. Fendley and E. Fradkin, *Topological order and conformal quantum critical points*, *Annals Phys.* **310** (2004) 493 [cond-mat/0311466] [SPIRES].

[6] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schrödinger symmetry*, *Phys. Rev.* **D 78** (2008) 046003 [arXiv:0804.3972] [SPIRES].

[7] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [arXiv:0804.4053] [SPIRES].

[8] W.D. Goldberger, *AdS/CFT duality for non-relativistic field theory*, *JHEP* **03** (2009) 069 [arXiv:0806.2867] [SPIRES].

[9] J.L.F. Barbon and C.A. Fuertes, *On the spectrum of nonrelativistic AdS/CFT*, *JHEP* **09** (2008) 030 [arXiv:0806.3244] [SPIRES].

[10] W.-Y. Wen, *AdS/NRCFT for the (super) Calogero model*, arXiv:0807.0633 [SPIRES].

[11] C.P. Herzog, M. Rangamani and S.F. Ross, *Heating up Galilean holography*, *JHEP* **11** (2008) 080 [arXiv:0807.1099] [SPIRES].

[12] J. Maldacena, D. Martelli and Y. Tachikawa, *Comments on string theory backgrounds with non-relativistic conformal symmetry*, *JHEP* **10** (2008) 072 [arXiv:0807.1100] [SPIRES].

[13] A. Adams, K. Balasubramanian and J. McGreevy, *Hot spacetimes for cold atoms*, *JHEP* **11** (2008) 059 [arXiv:0807.1111] [SPIRES].

[14] D. Minic and M. Pleimling, *Non-relativistic AdS/CFT and aging/gravity duality*, *Phys. Rev.* **E 78** (2008) 061108 [arXiv:0807.3665] [SPIRES].

[15] J.-W. Chen and W.-Y. Wen, *Shear viscosity of a non-relativistic conformal gas in two dimensions*, arXiv:0808.0399 [SPIRES].

[16] E.O. Colgain and H. Yavartanoo, *NR CFT_3 duals in M-theory*, arXiv:0904.0588 [SPIRES].

[17] A. Bagchi and R. Gopakumar, *Galilean conformal algebras and AdS/CFT*, arXiv:0902.1385 [SPIRES].

[18] A. Ghodsi and M. Alishahiha, *Non-relativistic D3-brane in the presence of higher derivative corrections*, arXiv:0901.3431 [SPIRES].

[19] A. Donos and J.P. Gauntlett, *Supersymmetric solutions for non-relativistic holography*, *JHEP* **03** (2009) 138 [arXiv:0901.0818] [SPIRES].

[20] S.S. Pal, *Anisotropic gravity solutions in AdS/CMT*, arXiv:0901.0599 [SPIRES].

[21] U.H. Danielsson and L. Thorlacius, *Black holes in asymptotically Lifshitz spacetime*, *JHEP* **03** (2009) 070 [arXiv:0812.5088] [SPIRES].

[22] M. Taylor, *Non-relativistic holography*, arXiv:0812.0530 [SPIRES].

[23] A. Adams, A. Maloney, A. Sinha and S.E. Vazquez, *$1/N$ effects in non-relativistic gauge-gravity duality*, *JHEP* **03** (2009) 097 [arXiv:0812.0166] [SPIRES].

[24] A. Akhavan, M. Alishahiha, A. Davody and A. Vahedi, *Non-relativistic CFT and semi-classical strings*, *JHEP* **03** (2009) 053 [arXiv:0811.3067] [SPIRES].

[25] M. Rangamani, S.F. Ross, D.T. Son and E.G. Thompson, *Conformal non-relativistic hydrodynamics from gravity*, *JHEP* **01** (2009) 075 [arXiv:0811.2049] [SPIRES].

[26] L. Mazzucato, Y. Oz and S. Theisen, *Non-relativistic branes*, *JHEP* **04** (2009) 073 [arXiv:0810.3673] [SPIRES].

[27] M. Schvellinger, *Kerr-AdS black holes and non-relativistic conformal QM theories in diverse dimensions*, *JHEP* **12** (2008) 004 [arXiv:0810.3011] [SPIRES].

[28] S. Sachdev and M. Mueller, *Quantum criticality and black holes*, arXiv:0810.3005 [SPIRES].

[29] S.A. Hartnoll and K. Yoshida, *Families of IIB duals for nonrelativistic CFTs*, *JHEP* **12** (2008) 071 [arXiv:0810.0298] [SPIRES].

[30] F.-L. Lin and S.-Y. Wu, *Non-relativistic holography and singular black hole*, arXiv:0810.0227 [SPIRES].

[31] D. Yamada, *Thermodynamics of black holes in Schrödinger space*, *Class. Quant. Grav.* **26** (2009) 075006 [arXiv:0809.4928] [SPIRES].

[32] C. Duval, M. Hassaine and P.A. Horvathy, *The geometry of Schrödinger symmetry in gravity background/non-relativistic CFT*, *Annals Phys.* **324** (2009) 1158 [arXiv:0809.3128] [SPIRES].

[33] P. Kovtun and D. Nickel, *Black holes and non-relativistic quantum systems*, *Phys. Rev. Lett.* **102** (2009) 011602 [arXiv:0809.2020] [SPIRES].

[34] S. Kachru, X. Liu and M. Mulligan, *Gravity duals of Lifshitz-like fixed points*, *Phys. Rev.* **D 78** (2008) 106005 [arXiv:0808.1725] [SPIRES].

JHEP07(2009)079

[35] K.-M. Lee, S. Lee and S. Lee, *Nonrelativistic superconformal M2-brane theory*, arXiv:0902.3857 [SPIRES].

[36] A. Galajinsky and I. Masterov, *Remark on quantum mechanics with $N = 2$ superconformal Galilean symmetry*, arXiv:0902.2910 [SPIRES].

[37] Y. Nakayama, M. Sakaguchi and K. Yoshida, *Non-relativistic M2-brane gauge theory and new superconformal algebra*, *JHEP* **04** (2009) 096 [arXiv:0902.2204] [SPIRES].

[38] Y. Nakayama, *Index for non-relativistic superconformal field theories*, *JHEP* **10** (2008) 083 [arXiv:0807.3344] [SPIRES].

[39] J. Gomis, K. Kamimura and P.K. Townsend, *Non-relativistic superbranes*, *JHEP* **11** (2004) 051 [hep-th/0409219] [SPIRES].

[40] J. Brugues, T. Curtright, J. Gomis and L. Mezincescu, *Non-relativistic strings and branes as non-linear realizations of Galilei groups*, *Phys. Lett.* **B 594** (2004) 227 [hep-th/0404175] [SPIRES].

[41] J. Kluson, *Non-relativistic non-BPS Dp-brane*, *Nucl. Phys.* **B 765** (2007) 185 [hep-th/0610073] [SPIRES].

[42] J. Gomis, K. Kamimura and P.C. West, *Diffeomorphism, kappa transformations and the theory of non-linear realisations*, *JHEP* **10** (2006) 015 [hep-th/0607104] [SPIRES].

[43] J. Gomis, K. Kamimura and P.C. West, *The construction of brane and superbrane actions using non-linear realisations*, *Class. Quant. Grav.* **23** (2006) 7369 [hep-th/0607057] [SPIRES].

[44] M. Sakaguchi and K. Yoshida, *Non-relativistic AdS branes and Newton-Hooke superalgebra*, *JHEP* **10** (2006) 078 [hep-th/0605124] [SPIRES].

[45] J. Gomis, F. Passerini, T. Ramirez and A. Van Proeyen, *Non relativistic Dp-branes*, *JHEP* **10** (2005) 007 [hep-th/0507135] [SPIRES].

[46] J. Gomis, J. Gomis and K. Kamimura, *Non-relativistic superstrings: a new soluble sector of $AdS_5 \times S^5$*, *JHEP* **12** (2005) 024 [hep-th/0507036] [SPIRES].

[47] E. Kiritsis and G. Kofinas, *Hořava-Lifshitz cosmology*, arXiv:0904.1334 [SPIRES].

[48] G. Calcagni, *Cosmology of the Lifshitz universe*, arXiv:0904.0829 [SPIRES].

[49] T. Takahashi and J. Soda, *Chiral primordial gravitational waves from a Lifshitz point*, arXiv:0904.0554 [SPIRES].

[50] G. Bonelli, A. Tanzini and M. Zabzine, *On topological M-theory*, *Adv. Theor. Math. Phys.* **10** (2006) 239 [hep-th/0509175] [SPIRES].